Effect of Aspect Ratio and Ply Orientation on Aeroelastic Response of Composite Plates

N. Sarigul-Klijn* and S. Oguz†
University of California, Davis, Davis, California 95616

Introduction

ACCURATE predictions of divergence and flutter behavior are important to assure that the flight vehicle is free of unstable modes within its mission velocities. Aeroelastic tailoring, a design process in which the main objective is reaching the minimum weight by positively utilizing the directional stiffness property of composite structures, has increasingly become an integral part of formal design procedure in flight structures.[1] As the major element of aeroelastic tailoring, structural and dynamic behaviors of laminated composite materials need to be understood.

Past studies have analyzed the effects of variation of laminate parameters, usually ply orientation or some other measure of bending-torsion coupling. An analytical investigation to determine the effects of bending-torsion stiffness coupling on divergence and flutter velocities of unswept, rectangular cantilevered graphite/epoxy plates were given in Ref. 2. By using a five-mode Rayleigh–Ritz formulation, the study showed that an unbalanced laminated composite wing, with negative stiffness coupling, experienced divergence, while positive coupling delayed the onset of flutter. They also experimentally showed that torsion and bending stall flutter occurred at high angles of attack, while divergence and bending-torsion flutter were critical at low angles of attack.

The same type of unbalanced laminate $[\theta_2/0]_s$, but for an entire range of -90 deg $\leq \theta \leq 90$ deg and using eight and nine modes, is analyzed in Ref. 3. The study showed that except for a small range around 0 and -90 deg, negative coupling causes a first-bending divergence, while positive coupling gives an infinite divergence velocity. The results are found to be in agreement with the experimental results of Ref. 2.

Unlike other studies, this paper investigates both flutter and divergence instabilities for the entire range of $[\pm\theta/0]_s$ type balanced laminate of moderate to high aspect ratio and includes eight and nine modes. The Rayleigh–Ritz method is chosen owing to its nature of identifying individual contributions of each mode into overall system response. A modified Theodorsen two-dimensional aerodynamic theory that accounts for parabolic deformation effects of the airfoil camber as derived by Spielberg in Ref. 4 is used. The results showing the effects of chordwise vibrations on the bending-torsion stiffness coupling as a function of aspect ratio and ply orientation angle are presented.

Analytical Model

The composite lifting surface is idealized as a rectangular plate in a uniform incompressible airflow in the direction of the negative chordwise y axis. The out of plane deflection of the plate in generalized coordinates is expressed as follows:

$$w = \sum_{i=1}^{n} \gamma_i(x, y) q_i(t) \tag{1}$$

where w is the lateral deflection, $q_i(t)$ is the generalized displacement of the ith mode, and $\gamma_i(x, y)$ is as follows:

$$\gamma_i(x, y) = \Phi(x) \cdot \Psi(y) \tag{2}$$

Three types of deformation mode shapes can be expressed as cantilevered beam modes for spanwise bending and torsion and free-free beam vibration modes for chordwise bending, and can be represented analytically by the following orthogonal functions:

$$\begin{aligned}
\Phi_{Bn}(x) &= C_n[\sin \beta_n x - \sinh \beta_n x - \alpha_n(\cos \beta_n x - \cosh \beta_n x)] \\
\Psi_{Bn}(y) &= 1 \\
\Phi_{Tm}(x) &= C_m \left[\sin \frac{(2m - 1)\pi y}{2L} \right], \qquad m = 0, 1 \ldots \\
\Psi_{Tm}(y) &= \frac{1}{2}\left(1 - \frac{2y}{C}\right) \\
\Psi_{Cp}(y) &= [\sin \beta_p y + \sinh \beta_p y + \alpha_p(\cos \beta_p y + \cosh \beta_p y)] \\
\Phi_{Cp}(x) &= \frac{C_p x}{L}\left(1 - \frac{x}{L}\right)
\end{aligned} \tag{3}$$

where β_n, α_n, β_p, and α_p are the mode shape function coefficients.[5]

Using the Rayleigh–Ritz method and assuming no structural damping, the aeroelastic equations of motion of the model are obtained in the form given as follows:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{Q_a\} \tag{4}$$

where $[M]$ and $[K]$ are $n \times n$ structural mass and stiffness matrices, which can be derived by taking the partial derivatives of kinetic- and strain-energy functions for symmetric laminate, respectively.[3] $\{Q_a\}$ is the generalized unsteady aerodynamic force vector, and $\{q\}$ is the vector of the generalized coordinates. The unsteady aerodynamic model is based on thin airfoil assumptions. Spielberg[4] derived relationships for the unsteady aerodynamic lift and moment coefficients for rigid-body bending, rigid-body torsion, and parabolic deformation of the airfoil camber as a function of reduced frequency and elastic axis position when the airfoil oscillations are of simple harmonic. The generalized modal aerodynamic force for simple harmonic motion is

$$Q_i(t) = \pi\rho\omega^2 b^2 e^{i\omega t} \sum_{j=1}^{n} A_{ij} q_j \tag{5}$$

where A_{ij} are the elements of the $n \times n$ aerodynamic influence matrix that has a complex form because of aerodynamic damping.[3]

By using the same harmonic motion expressions for the generalized displacements, the equations of motion for flutter analysis are expressed in the form given next:

$$([K] - \omega^2[B])\{\bar{q}\} = 0 \quad \text{with} \quad [B] = [M] + \pi\rho b^2[A] \tag{6}$$

Because of the complex incompressible aerodynamic terms, $[B]$ matrix contains complex terms. Among the various solution techniques, the U-g method is used. By assuming a single-value artificial damping of g, the damping coefficient is intro-

Received Sept. 16, 1996; revision received Dec. 12, 1997; accepted for publication Feb. 2, 1998.

*Co-Director and Professor, Transportation Noise Control Center, Department of Mechanical and Aeronautical Engineering. Senior Member AIAA.

†Graduate Student, Mechanical and Aeronautical Engineering; currently Project Engineer, OEA Aerospace Inc., Fairfield, CA 95619.

duced in the system by multiplying $[K]$ by $(1 + ig)$. This gives an eigenvalue problem in the form

$$([B] - z[K])\{\bar{q}\} = O \quad \text{with} \quad z = (1 + ig)/\omega^2 \tag{7}$$

The eigenvalue problem shown in Eq. (7) is solved for U and g values by assuming a wide range of reduced frequencies. Flutter and divergence velocities and the modes associated with a particular instability are then determined.

Numerical Investigation and Discussion of Flutter and Divergence Results

A multilayered T300/5208 graphite/epoxy plate model is used in the numerical studies. The analysis was performed with eight (four bending and four torsional), and nine (four bending, four torsional, and one chordwise bending) modes for aspect ratios of 2, 4, 6, and 8. The entire range of -90 deg $< \theta <$ 90 deg of $[\pm\theta/0]_s$ balanced laminate was studied. For each case the five lowest modes were analyzed with the U-g plot to determine the flutter and divergence velocities.

It is shown that the inclusion of Chordwise Vibration Mode (CVM) has a significant influence on the amount of bending-torsion coupling. For example, for aspect ratio 4, a very strong bending-torsion coupled flutter mode is observed for ply orientation angle ranges of -70 deg $< \theta < -50$ deg and 50 deg $< \theta <$ 70 deg. Figure 1 shows this effect for $[60/0]_s$ laminate. Without CVM it is observed that second bending mode becomes unstable with the first torsion mode coming close to the flutter region and causing somewhat coupled flutter. However, the presence of CVM switches the flutter from second bending to first torsion and also totally stabilizes the second bending branch, causing a lower flutter speed. On the other hand, for $[\mp 60/0]_s$ laminate as opposed to $[\pm 60/0]_s$ laminate, inclusion of CVM increases the flutter speed by switching the flutter mode from torsion to bending. As the aspect ratio gets higher, inclusion of CVM affects the flutter speed but not the flutter mode (Fig. 2). For aspect ratio 8 and the $[\pm 60/0]_s$ model, for example, the flutter speed is reduced from 39.4 m/s to 36.6 m/s, but the flutter mode remained as the first torsion.

The implications of the bending-torsion coupling and the influence of CVM on divergence boundaries were also studied. Naturally, the divergence velocities showed an opposite trend to flutter. It was observed that $[\pm\theta/0]_s$ laminate always exhibited a first bending divergence. Therefore, it is accurate to indicate that the influence of CVM on divergence is directly related to the influence on the first bending mode. First bending mode is usually a very fast converging mode and is not influenced by the inclusion of CVM. However, a portion be-

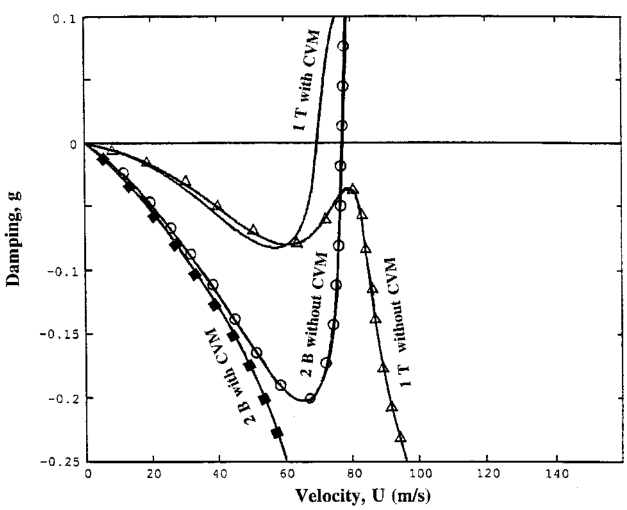

Fig. 1 ***U*-*g* diagram for $[\pm 60/0]_s$ balanced laminate, aspect ratio = 4.**

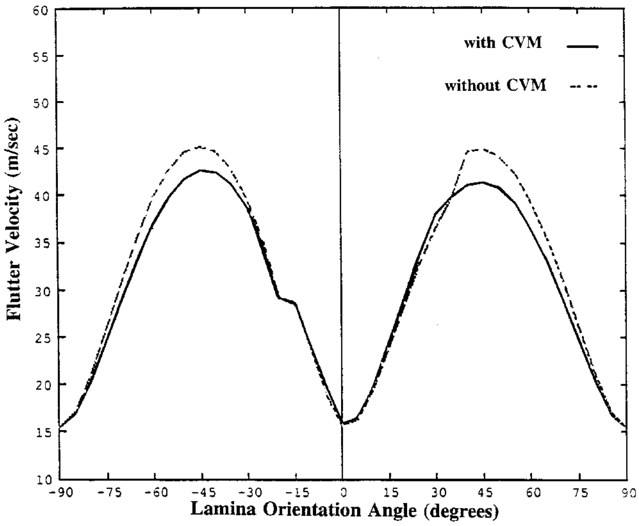

Fig. 2 Effects of chordwise modes on flutter velocities as ply orientation angle changes, aspect ratio = 8.

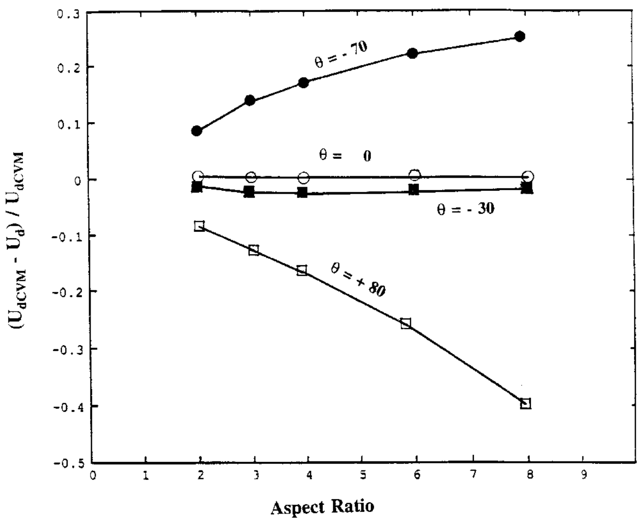

Fig. 3 Percentage increment in divergence speed U_d as a result of the inclusion of CVMs as a function of aspect ratio.

tween -90 deg $< \theta < -45$ deg and 75 deg $< \theta <$ 90 deg is affected by the inclusion of CVM. This affect increases as the aspect ratio gets larger. Unlike the flutter velocity, a trend was observed on the affect of the CVM on divergence. Figure 3 shows that the influence of CVM on divergence within the range of the lamina angles, indicated earlier, dramatically increases as the aspect ratio increases. However, a similar general conclusion cannot be drawn for flutter because the influence of CVM on flutter is more complicated and involves different mode types for the same laminate with different aspect ratios.

Another point is that divergence might be eliminated for a certain range of $[\pm\theta/0]_s$ laminates. The increase in aspect ratio widens the divergence-free region. It seems that for high-aspect-ratio lifting surfaces, a laminate design with positive top and bottom angles is very reasonable because the sign does not affect the flutter velocity and divergence is eliminated. This general conclusion cannot be drawn for smaller aspect-ratio wings because the opposite trends between the divergence and flutter boundaries become much more substantial.

Concluding Remarks

Tailoring studies were performed showing the effects of chordwise vibration mode on the flutter and divergence behavior of moderate- to high-aspect-ratio balanced laminated lifting surfaces as a function of ply angles. The numerical results indicate the strong influence of CVM inclusion in aeroelastic instability computations. With the inclusion of CVM, a better convergence in mode shapes is obtained regardless of the aspect-ratio effects. For moderate-aspect-ratio lifting surfaces the inclusion of CVM switches the flutter mode from second bending to first torsion, or from first torsion to second bending as a function of laminate orientation angles. At high-aspect-ratio lifting surfaces the inclusion of CVM affects the flutter speed but does not influence the flutter mode type. In divergence the inclusion of CVM becomes more critical within certain ply angles for lifting surfaces of aspect ratios larger than 6. For high-aspect-ratio lifting surfaces, a laminate design with positive top and bottom angles is recommended.

The results presented would be very helpful in design, particularly at the initial design phase to size the lifting surface to achieve mission velocities of the flight vehicle to be free of aeroelastic instabilities.

References

[1]Shirk, M., Hertz, T. J., and Weisshaar, T. A., "Aeroelastic Tailoring-Theory, Practice and Promise," *Journal of Aircraft*, Vol. 23, No. 1, 1986, pp. 6–18.

[2]Hollowell, S. J., and Dugundji, J., "Aeroelastic Flutter and Divergence of Stiffness Coupled, Graphite/Epoxy Cantilevered Plates," *Journal of Aircraft*, Vol. 21, No. 1, 1982, pp. 69–76.

[3]Oguz, S., "Aeroelastic Tailoring of a Composite Wing for Flutter and Divergence Control," M.S. Thesis, Univ. of California, Davis, CA, 1995.

[4]Spielberg, I. N., "The Two Dimensional Incompressible Aerodynamic Coefficients for Oscillatory Changes in Airfoil Camber," *Journal of Aeronautical Sciences*, Vol. 1, No. 6, 1953, pp. 432–436.

[5]Rao, S. S., *Mechanical Vibrations*, Addison–Wesley, New York, 1990, pp. 125–135.

Multiple Attractors in Inertia-Coupled Velocity-Vector Roll Maneuvers of Airplanes

Anirudh Modi* and N. Ananthkrishnan†

Indian Institute of Technology, Bombay, Mumbai 400076, India

Nomenclature

I_x, I_y, I_z = roll, pitch, and yaw inertia, respectively
i_1, i_2, i_3 = cyclically $(I_z - I_y)/I_x$, etc.
l, m, n = roll, pitch, and yaw moment coefficients, respectively
p, q, r = roll, pitch, and yaw rates, respectively
y, z = side and normal force coefficients, respectively
α, β = angle of attack and sideslip, respectively
δa, δe, δr = aileron, elevator, and rudder deflection, respectively

Subscripts
α, p, ... = stability derivative with respect to α, p, ...

Superscripts
$\hat{}$ = division by either i_1, i_2, or i_3
$'$ = transpose

I. Introduction

INERTIA-coupled rapid roll maneuvers of aircraft involve nonlinear dynamics that tend to result in excessive sideslip and undesirable pitching motion, and that need to be avoided to prevent instability and the loss of pilot control.[1] In a roll maneuver initiated by an aileron deflection, this calls for use of the rudder to counter the sideslip, and elevator deflection to control the pitching motion. However, the nonlinear nature of the problem makes it difficult to prescribe the required rudder and elevator deflections for a given aileron deflection or roll rate demand.

Significant progress in the solution of the problem of inertia-coupled rapid rolls could be achieved only after the instability was identified with a jump phenomenon. Schy and Hannah[2] showed that large sideslips and pitch rates are created when the system jumps at a saddle-node bifurcation point from one attractor (stable solution) to another with typically large values of roll rate, pitch rate, and sideslip. They used the customary fifth-order pseudosteady-state (PSS) approximation[3] with a linear aerodynamic model, and aileron deflection as the parameter. In the absence of a nonlinear aerodynamic model in their analysis, the jump phenomenon could be attributed solely to the effect of the nonlinear inertia coupling terms. Their study was extended to include nonlinear aerodynamic effects by Young et al.[4] In recent years, continuation methods have been used by Jahnke and Culick[5] to evaluate jump in rapid roll maneuvers for the F-14 with a nonlinear aerodynamic model.

On the basis of Schy and Hannah's work,[2] the problem of limiting sideslip and pitch rate in inertia-coupled roll maneuvers could be restated as the equivalent problem of avoiding the saddle-node bifurcation and multiple attractors in the PSS formulation. Carroll and Mehra[6] described a nonlinear variation of the rudder deflection as a function of the aileron deflection, called an aileron–rudder interconnect (ARI), that would avoid jump. Ananthkrishnan and Sudhakar[7] devised a linear ARI strategy to avoid the saddle-node bifurcation by using the idea of tuning a perturbation parameter across a transcritical bifurcation. Inertia-coupled roll maneuvers with a zero-sideslip constraint, called coordinated rolls, were studied by Ananthkrishnan and Sudhakar.[8] Coordinated rolling was seen to avoid multiple attractors, and thus prevent jump, but this required a nonlinear ARI relationship.

Velocity-vector roll (VVR) maneuvers, where the angular velocity vector coincides with the flight path (linear velocity vector), have been considered by designers as a possible strategy to limit sideslip and pitch rate in rapid roll maneuvers of aircraft.[9] The aim of this Note is to investigate whether a VVR strategy eliminates multiple attractors and prevents jump in rapid roll maneuvers.

II. Pseudosteady-State Analysis

The fifth-order PSS equations in the five variables (p, q, r, β, and α) are solved with the linear aerodynamic model of aircraft B of Schy and Hannah.[2] The use of a linear aerodynamic model ensures that the nonlinear behavior is only a result of the inertia coupling nonlinearities. The inclusion of nonlinear aerodynamic terms to this model is expected to provide better quantitative values at large angles of attack and sideslip, while maintaining an unchanged qualitative picture. In particular, for the limited aim of this study, the presence of multiple attractors is expected to be unaffected by the nonlinear aerodynamics. In fact, it is noticed that the numerical values for jump onset obtained with the linear aerodynamic model are fairly accurate as long as the saddle-node bifurcation point occurs for small angles of attack and sideslip.

Received June 19, 1997; revision received Nov. 11, 1997; accepted for publication March 27, 1998.

*Undergraduate Student, Department of Aerospace Engineering; currently Graduate Student, Aerospace Engineering, Pennsylvania State University, University Park, PA 16802.

†Assistant Professor, Department of Aerospace Engineering.